## Mass transfer between a liquid and a binary rotating/ fixed disc system in a closed cylinder

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The electrochemical method is applied to the determination of mass transfer coefficients between a liquid and opposite circular discs, one of which is rotating, enclosed by a cylinder. Local measurements made at the fixed disc confirm flow schemes proposed in the literature. The global mass transfer coefficients are correlated empirically and compared with a literature correlation for nonelectrochemical data.

#### Nomenclature

- $C_{\rm r}$  radial confinement ratio,  $C_{\rm r} = R_{\rm T}/R_{\rm F}$
- $C_{\rm v}$  vertical confinement ratio,  $C_{\rm v} = H/R_{\rm T}$
- $C_{\infty}$  concentration of ferricyanide ions (mol m<sup>-3</sup>)
- D molecular diffusion coefficient of ferricyanide ions  $(m^2 s^{-1})$
- F Faraday constant (96 500 A s mol<sup>-1</sup>)
- H distance between the discs (m)
- $i'_{\rm L}$  local limiting current density (A m<sup>-2</sup>)
- $\overline{k}_{d}$  average mass transfer coefficient (m s<sup>-1</sup>)
- $k'_{\rm d}$  local frictional mass transfer coefficient (m s<sup>-1</sup>)
- $k_{\rm d}(r)$  local mass transfer coefficient (m s<sup>-1</sup>)
- N rotating disc velocity (r.p.m.)
- *r* radial coordinate (m)
- R radius of disc (m)
- $Re_{\rm H}$  Reynolds number based on  $H (= \omega H^2 / \nu)$
- <u>*Re*</u><sub>R</sub> Reynolds number based on  $R (= \omega R^2 / \nu)$
- $\overline{\mathit{Sh}}_{\mathrm{H}}$  average Sherwood number based on H

## 1. Introduction

In industrial applications, the rotation of a circular disc at a given distance from an opposite stationary disc, which is generally the bottom of a cylindrical cavity, is frequently encountered, and many theoretical and experimental studies can be found in the literature. A number of theoretical and experimental works have also dealt with unconfined systems (i.e., with discs in infinite media).

The rotation of an enclosed disc has been considered in papers dealing with flow structure [1-4], with heat exchange between the fluid and the two (rotating and fixed) discs [5], or with mass exchange [6] in view of the application, by analogy, to heat transfer.

Only a few previous works have used a liquid as the fluid. Hudson *et al.* [3, 7] and Matsuda [8] considered such a fluid; in view of electroanalytical applications, the latter theoretically examined ionic mass transfer to a fixed disc located in an infinite rotating liquid. Mass transfer between a dialysis membrane and a rotating

- $(=\overline{k}_{\rm d}H/D)$
- $\overline{Sh}_{R}$  average Sherwood number based on R (=  $\overline{k}_{d}R/D$ )
- s wall velocity gradient  $s^{-1}$
- Sc Schmidt number (=  $\nu/D$ )

## Greek

- $\overline{\epsilon}$  mean porosity
- $\nu$  kinematic viscosity (m<sup>2</sup> s<sup>-1</sup>)
- $\nu_{\rm e}$  number of electrons in the electrode reaction
- $\rho$  density, (kg m<sup>-3</sup>)

 $\omega$  angular velocity (s<sup>-1</sup>)

## Subscripts

- F at the fixed disc
- T at the rotating disc

## Exponent

at the disc rotating in an infinite media

liquid has also been theoretically [9, 10] and experimentally studied [9, 11]. The so-called 'Pump Cell' also used an enclosed rotating disc [12,13] but, due to the presence of an axial flow through the opposite fixed disc, the resulting hydrodynamic situation in this cell cannot be compared to the situation where such a flow does not exist.

To the best of our knowledge, the electrochemical method for the measurements of wall to liquid mass transfer coefficients has not yet been applied to systems involving rotating and stationary opposite discs. For example, Lehmkuhl and Hudson [7] used the rather imprecise method of cinnamic acid dissolution for the determination of mass transfer coefficients between a liquid and a disc rotating above the bottom of a cylindrical container.

Within the scope of research dealing with a system involving several rotating and stationary discs [14], the electrochemical method has been applied only to the case of two discs (one rotating, one stationary). The main objective was to obtain complementary, and eventually more precise, results than those of [7] in a comparable hydrodynamic flow regime. In a theoretical development concerning mass transfer between a liquid and a fixed disc located at a small distance from a geometrically identical disc rotating in conditions of laminar flow, Arvia and Marchiano [15] concluded that the kinematic viscosity and the molecular diffusion coefficient could be simultaneously determined from experimental data. The present work seemed to be an opportunity to test such a conclusion.

## 2. Experimental aspects

The cell (Fig. 1(a)) was adapted from that used in [16] where a disc rotated at a small distance from a porous disc, thereby leading to a self-pumping operation. Made of altuglas, the cell was cylindrical (0.06 m interior diameter) and consisted of four parts. Between parts A and B, was an altuglas disc, C, in which a nickel disc was inserted (radius  $R_{\rm F}$ ). Through the cover D, of B, the cylindrical element, E, was moved vertically by screwing; this element supported the rotating component of a Tacussel-EDI system, at the lower part of which a circular disc of nickel was mounted (radius  $R_{\rm T}$ ). For experiments in an enclosed cylinder  $(R_T \neq R_F)$ , an annular cylinder (outer diameter 0.06 m; inner and hole diameter,  $R_{\rm F}$ ) made of altuglas was located in (A) and over (C). The general configuration of the system is given in Figure 1(b).

The distance [H: 1;2;3;6;9;12 or 17 mm] between the rotating (radius  $R_{\rm T}$ ) and the fixed (radius  $R_{\rm F}$ ) discs was varied by moving part E. The rotating disc velo-

city, N, was varied from 1.3 to 200 r.p.m.. Except in the case of local measurements (see Fig. 1(c) and 5(a)),  $R_{\rm F}$  had several values: 0.0075, 0.01, 0.015 and 0.02 m, while  $R_{\rm T}$  was approximately equal to  $R_{\rm F}$ ( $R_{\rm T} \sim 0.99 R_{\rm F}$ ).

The mass transfer coefficient between the liquid and the discs was measured electrochemically using the classical technique of reduction of potassium ferricyanide ions in an alkaline medium. The electrical circuit was a three electrode (both discs and a nickel wire working as a reference electrode) potentiostatic circuit using a Tacussel-PRT 20-2 potentiostat. The liquid was a mixture of 0.005 M Fe(CN)<sub>6</sub><sup>-3</sup> and 0.05 M $Fe(CN)_6^{-4}$  in 0.5 M aqueous sodium hydroxide; the true ferricyanide concentration was periodically determined by amperometric titration with a cobalt salt using a platinum rotating disc electrode. The liquid was maintained at 30 °C by partial immersion of the cell in a thermostatically controlled water bath. At 30 °C, the electrolyte properties were: den- $\rho = 1050 \,\mathrm{kg}\,\mathrm{m}^{-3};$ sity, kinematic viscosity,  $\nu = 9.4 \times 10^{-7} \text{m}^2 \text{s}^{-1}$ ; molecular diffusion coefficient of ferricyanide,  $D = 8.8 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$  and Schmidt number,  $Sc = \nu/D = 1070$ .

The radial flow confinement was quantified by the ratio  $C_{\rm r} = R_{\rm T}/R_{\rm F}$  (varied from 0.25 to 0.99), while the vertical flow confinement was represented by  $C_{\rm v} = H/R_{\rm T}$  (varied from 0.1 to 2.6).

Two categories of information were derived from the mass transfer measurements to the fixed disc: (i) the spatial and time-averaged mass transfer coefficient,  $\overline{k}_d$  and (ii) the local values of the wall shear stress. These local values were obtained by means of



Fig. 1. Schematic sketch of the experimental equipment.



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21 circular (diameter  $400 \,\mu$ m) nickel microelectrodes inserted in an altuglas disc (diameter  $0.06 \,\text{m}$ ), and regularly distributed (2 mm between two neighbouring centres) as in [17] along two perpendicular radii  $R_1$  and  $R_2$  (see Fig. 1(b)). Such a technique using microelectrodes embedded 'in an inert wall' for measurements of local friction is well known. The local limiting current density,  $i'_{\rm L}$ , and thus the local frictional mass transfer coefficient  $k'_d = i'_{\rm L}/(\nu_{\rm e}Fc_{\infty})$ , is known to be proportional to  $s^{1/3}$ , where s is the wall velocity gradient [18]. In the present paper the experimental data are given in the form of  $k'_{\rm d}$ .

## 3. Results

# 3.1. Mass transfer between the liquid and the whole surface of the discs.

3.1.1. Fixed disc. The spatial mean mass transfer coefficients were measured for  $C_r = 0.99$  (maximum radial confinement) with different discs, and  $C_v$  varied. It is observed (Fig. 2) that  $\bar{k}_d$  is proportional to  $N^{1/2}$  (N is the rotational velocity in r.p.m.) or  $\omega^{1/2}$ , when end effects do not affect a high fractional surface area of the disc. An attenuation of such effects may occur if the disc diameter is large or if the distance between the discs, H, is small. Such a proportionality  $\bar{k}_d \propto \omega^{1/2}$  means that the Sherwood number  $S\bar{h}_{\rm H} = \bar{k}_d H/D$  is proportional to  $Re_{\rm H}^{1/2} = (\omega H^2/\nu)^{1/2}$ , or that the ratio  $S\bar{h}_{\rm H}/Re_{\rm H}^{1/2}$  is constant. This ratio remains unchanged if a characteristic dimension other than H is chosen  $(R_{\rm T} \text{ or } R_{\rm F} \text{ for example})$ . Plots of  $S\bar{h}_{\rm H}/(Re_{\rm H})^{1/2}$  against  $Re_{\rm H}$  are given in Fig.

Plots of  $Sh_{\rm H}/(Re_{\rm H})^{1/2}$  against  $Re_{\rm H}$  are given in Fig. 3(a)–(c). It can be seen that for a given value of the ratio  $H/R_{\rm T}$ ,  $\overline{Sh}_{\rm H}/(Re_{\rm H})^{1/2}$  increases with  $Re_{\rm H}$  up to a limiting value. The higher  $H/R_{\rm T}$ , the higher is the  $Re_{\rm H}$  value at which the limiting value is approximately



Fig. 2. Experimental variation (examples) of  $\bar{k}_d$ , at a fixed disc, as function of  $N_{1/2}$ .  $R_{\rm T}$ /mm,  $C_{\rm v}$ : ( $\odot$ ) 20, 0.1; ( $\bullet$ ) 15, 0.8; ( $\bigtriangledown$ ) 10, 1.7; ( $\checkmark$ ) 7.5, 2.26.  $C_{\rm r} = 0.99$ .

reached; a vertical dotted line indicates the position of this  $Re_{\rm H}$  value. It can be considered that at the left of each dotted line the increase in  $Re_{\rm H}$  through the increase of the rotation velocity progressively organizes the flow near the fixed disc; in other words the flow in the cylindrical system is only organized for  $Re_{\rm H}$  values beyond the dotted line. In Fig. 3(c), no limiting value is reached for  $H/R_{\rm T}=2.26$  in the explored rotating velocity range.

The mean limiting values deduced from Fig. 3(a)-(c) are shown (Fig. 3(d)) to vary empirically as  $(H/R_T)^{-0.28}$ , and finally, the ensemble of the results corresponding to organized flows in the system (right of the dotted lines in Fig. 3(a)-(c)) are seen to be described (Fig. 4) by the following empirical correlation:

$$\overline{Sh}_H = 0.2 (H/R_T)^{-0.28} Re_H^{1/2} Sc^{1/3}$$
(1)

with a correlation coefficient of 0.88. Expression 1 applies for  $6 \le Re_{\rm H} \le 7000$  when  $0.1 \le H/R_{\rm T} \le 1$  and for  $Re_{\rm H}^* < Re_{\rm H} < 7000$  when  $1 < H/R_{\rm T} \le 1.7$ , with  $Re_{\rm H}$  empirically depending on  $H/R_{\rm T}$  as

$$\log_{10} Re_{\rm H}^* = 1.81 + 0.92 (H/R_{\rm T})$$

This domain of validity for Equation 1 corresponds to a globally laminar flow unmodified by end effects.

3.1.2. Rotating disc. The geometrical conditions were the same as those used in mass transfer measurements to the fixed discs ( $C_r = 0.99$ , different values of  $C_v$ ). The mean mass transfer coefficient  $\bar{k}_d$ varies as  $N^{1/2}$  for some cases; for other cases, there is an attenuation for high values of N.

The data analysis reveals a negligible influence of  $C_v = H/R_T$  as  $(H/R_T)^{0.04}$ , thus allowing to write the empirical equation:

$$\overline{Sh}_H = 0.655 Re_H^{0.52} Sc^{1/3} \tag{2}$$

valid for  $6 \le Re_{\rm H} \le 7000$  and  $0.12 \le C_{\rm v} \le 2.2$ , with a correlation coefficient of 0.993. If the exponent of the Reynolds number is taken equal to be 0.5, the following empirical correlation is obtained, with  $R_{\rm T}$  as the characteristic dimension:

$$\overline{Sh}_{R_{\tau}} = 0.84 R e_{R_{\tau}}^{0.5} S c^{1/3} \tag{3}$$

with a correlation coefficient of 0.91.

#### 3.2. Local shear stress over the fixed disc

The local shear stress measurements were made for two confinement situations previously considered by other authors [7] and schematically shown in Fig. 5(a) and (b).

For the case of Fig. 5(a), which corresponds to a vertical confinement, radial distributions of  $k_d'$  such as those given for three rotating velocities, H=0.001 m and  $C_r=0.25$  (with  $R_T=0.0075 \text{ m}$  and  $R_F=0.03\text{m}$ ) were obtained. A minimum of  $k_d'$  at the centre of the disc is observed, and two symmetrical maxima approximately located below the edge of



**Fig. 3.** Mass transfer at a fixed disc: [(a);(b);(c)] a few examples of variations of  $\overline{Sh}_H/Re_H^{1/2}$  with  $Re_R$  and [(d)] graphical determination of the influence of  $H/R_T$ . (a) with H = 6 mm and  $H/R_T$ : ( $\bigcirc$ ) 0.8, ( $\bullet$ ), 0.4, ( $\bigtriangledown$ ) 0.3; (b) with H = 12 mm and  $H/R_T$ : ( $\bigcirc$ ) 1.6, ( $\bullet$ ) 1.2, ( $\bigtriangledown$ ) 0.8, ( $\bullet$ ) 0.6; (c) with H = 17 mm and  $H/R_T$ : ( $\bigcirc$ ) 2.26, ( $\bullet$ ) 1.7, ( $\bigtriangledown$ ) 1.13, ( $\bullet$ ) 0.85.

the rotating disc, the size of which is shown in the figure. Beyond each maximum, there is a progressive decrease of  $k_d'$  down to low values corresponding to zones with small radial velocity gradients. The form of such distributions of  $k_d'$  agree with the flow schematically represented at the top of Fig. 5(a).

For radial confinement ( $C_r = 0.99$ ), Fig. 5(b) shows results obtained for three values of H and one rotating velocity. In spite of an obvious dispersion probably due to end effects, the distributions correspond to the schematic flow representation of Fig. 5(b).

Finally, both categories of results of Fig. 5(a) and (b) confirm the flow schemes proposed by Lemkuhl and Hudson [7] from qualitative observations of the flow of a coloured tracer.

The idea of Arvia and Marchiano [15] mentioned in the introduction is very interesting. The theoretical calculations of the mass transfer coefficient between a liquid in laminar flow and a fixed disc situated at a small distance H from a rotating disc having the



Fig. 4. Mass transfer correlations for transfer to the fixed disc.



Flow schemes [7],  $R_{\rm T}/R_{\rm F} < 0.8$ 



Fig. 5. Radial distributions of the local frictional transfer coefficient,  $k_d'(r)$ , for an enclosed fixed disc configuration: (a) vertical confinement only, and (b) vertical and radial confinement. Key for (a): (...) calculated from Equation 6; ( $\bigcirc$ ) N = 1.3, ( $\bigtriangledown$ ) N = 6.5 and ( $\bullet$ ) N = 13 r.p.m. with  $R_T = 0.0075$  m,  $R_F = 0.03$  m and H = 0.001 m. Key for (b): ( $\bigcirc$ ) H = 1, ( $\bullet$ ) H = 2 and ( $\bigtriangledown$ ) H = 3 mm for rotating disc with  $C_r = 0.99$  and N = 16 r.p.m.

| Authors      | Correlation   | Validity  | Method                                 | System     |
|--------------|---|---|--|------------|
| Matsuda [8]  | $\overline{Sh}_{R_{\rm F}} = 0.761 Re_{R_{\rm T}}^{1/2} Sc^{1/3}$   | _   | theoretical                            | $\bigcirc$ |
| Colton [10]  | $\overline{Sh}_{\rm R_F} = 0.28 Re_{\rm R_T}^{0.567} Sc^{1/3}$  | $H/R_{\rm T} = 0.112; \ R_{\rm T}/R_{\rm F} = 0.9$<br>$8 \times 10^3 < Re_{\rm P} \leq 3.2 \times 10^4$   | dialysis                               |            |
|              | $\overline{Sh}_{R_F} = 0.0443 \ Re_{R_T}^{0.746} Sc^{1/3}$  | $3.2 \times 10^4 \leq Re_{R_T} < 8.2 \times 10^4$   | dialysis                               |            |
|              | $\frac{Sh_{\rm R_{\rm F}}}{Sh_{\rm R_{\rm F}}} = 0.5376 \ Re_{\rm R_{\rm T}}^{1/3} Sc^{1/3}$ $\overline{Sh}_{\rm R_{\rm F}} = 0.0262 \ Re_{\rm R_{\rm T}}^{0.8} Sc^{1/3}$ | $Re_{R_{T}} < 3 \times 10^{4}$ (laminar)<br>$Re_{R_{T}} > 3 \times 10^{4}$ (turbulent)                    | dialysis/theoret.<br>dialysis/theoret. |            |
| Leonard [11] | $\overline{Sh}_{R_{\rm F}} = 0.134 \ Re_{R_{\rm T}}^{0.68} Sc^{1/3}$  | $\begin{array}{l} 387 < Sc < 1622 \\ 5.8 \times 10^3 < Re_{R_{\mathrm{T}}} < 1.1 \times 10^5 \end{array}$ | dialysis                               |            |
| Present work | $\overline{Sh}_{R_{\rm F}} = 0.2 (H/R_{\rm T})^{-0.28} Re_{R_{\rm T}}^{0.5} Sc^{1/3}$   | $0.1 \leq H/R_{\rm T} \leq 1.7$ $87 \leq Re_{\rm R_T} \leq 9 \times 10^3$                                 | electrochemical                        | )          |

Table 1. Empirical and/or theoretical correlations for mass transfer at the enclosed fixed disc.

same diameter, is possible using the velocity distribution proposed by Gruhne [19] or by Pecheux [20]. The calculation effectively leads to [15]

$$\overline{k}_d^{-1} = 0.222 \, (H/D) + 2.28 \, \nu^{1/4} \omega^{-1/2} D^{-3/4} \quad (4)$$

where  $\omega$  is the angular velocity of the rotating disc. As outlined in [15], it follows from Expression 4 that the plotting of  $\overline{k}_{d}^{-1}$  against  $\omega^{-1/2}$  is linear and leads simultaneously to both *D* and  $\nu$ . Unfortunately, the theoretical development of [15] assumes implicitely that the disc surface is uniformly accessible, that is, that the distribution of the local mass transfer coefficients is uniform.

It was not possible to test experimentally such a hypothesis in the present work because only local wall velocity gradients (i.e., local wall shear stress), were measured. Figure 5(a) and (b) shows that the spacial distribution of these gradients or stress is not uniform. Plots of  $\bar{k}_d^{-1}$  against  $\omega^{-1/2}$  for mass transfer

results concerning the fixed disc never led to a straight line, a conclusion which contradicts Expression 4. This agrees with the theoretical conclusions of Matsuda [8] for mass transfer at a fixed disc of radius  $R_{\rm T}$  situated in a uniformly rotating fluid. The calculations of Matsuda led to a nonuniform distribution of the local mass transfer coefficients,  $k_{\rm d}$ , over the disc;  $k_{\rm d}$  depends on the radial position r as

$$k_d(r) = 0.761 \ D^{2/3} \nu^{-1/6} \omega^{1/2} \frac{r}{\left(R_T^3 - r^3\right)^{1/3}}$$
(5)

According to this theoretical equation,  $k_d(r)$  is zero at the centre of the disc and increases with r up to an infinite value at the disc periphery  $(r = R_T)$ .

The distribution  $k_d'(r)$  has been calculated for N=13 r.p.m., from the wall velocity gradient s(r) deduced from the theoretical velocity distributions [19, 20] and using the following expression:

Table 2. Empirical and/or theoretical correlations for mass transfer at the enclosed rotating disc.

| Authors                 | Correlation   | Validity   | Method             |
|-------------------------|---|--|--------------------|
| Lehmkuhl and Hudson [7] | $\overline{Sh}_{R_{T}} = 0.196 (H/R_{T})^{0.26} Re_{R_{T}}^{0.63} Sc^{1/3}$   | $\begin{array}{l} 3.74 \times 10^{-3} < H/R_{\rm T} < 0.33 \\ 0.314 < R_{\rm T}/R_{\rm F} < 0.575 \\ 2.9 \times 10^3 < Re_{\rm R_{\rm T}} < 1.28 \times 10^5 \end{array}$  | dissolution        |
| Daily and Nece [2]      | $ \frac{\overline{Sh}_{R_{\rm T}}}{\overline{Sh}_{R_{\rm T}}} = 0.04 (H/R_{\rm T})^{-0.16} Re_{R_{\rm T}}^{0.75} Sc^{1/3}  \overline{Sh}_{R_{\rm T}} = 0.05 (H/R_{\rm T})^{0.1} Re_{R_{\rm T}}^{0.8} Sc^{1/3}  \overline{Sh}_{R_{\rm T}} = 0.05 (H/R_{\rm T})^{0.075} Re_{R_{\rm T}}^{0.8} Sc^{1/3} $ | $\begin{split} R_{\rm T}/R_{\rm F} &\approx 1; \ 6\times 10^4 < Re_{\rm R_T} < 4\times 10^6 \\ R_{\rm T}/R_{\rm F} &\approx 1; \ Re_{\rm R_T} > 8.4\times 10^4 \\ R_{\rm T}/R_{\rm F} &\approx 1; \ Re_{\rm R_T} > 8.4\times 10^4 \\ 0.0127 < H/R_{\rm T} < 0.217 \end{split}$ | torque measurement |
| Soo and Princeton [1]   | $\overline{Sh}_{R_{T}} = 0.0122 (H/R_{T})^{-0.25} Re_{R_{T}}^{0.75} Sc^{1/3}$   | $0.125 \ge H/R_{\rm T} \ge 5 \times 10^{-3}$<br>3.14 × 10 <sup>4</sup> < $Re_{\rm R_{\rm T}} < 4.2 \times 10^{7}$  | theoretical        |
| Grunow [21]             | $\overline{Sh}_{R_T} = 0.0155 \ Re_{R_T}^{0.8} Sc^{1/3}$  | $R_{\rm T}/R_{\rm F} = 0.97; \ Re_{\rm R_T} > 3 \times 10^5$<br>$H/R_{\rm T} = 0.0199$   | theoretical        |
| Present work            | $\overline{Sh}_{R_{T}} = 0.85 \ Re_{R_{T}}^{0.5} Sc^{1/3}$  | $R_{\rm T}/R_{\rm F} pprox 1; \ 0.1 < H/R_{\rm T} < 2.26$<br>87 < $Re_{ m R_T} < 9700$   | electrochemical    |



Fig. 6. Mass transfer correlations for transfer to the fixed disc. Comparison with the literature. Key: (a) [10] theoretical, (b) [11], (c) [10] experimental, (d) this work.

$$k'_d(r) = 0.863 \ \frac{D^{2/3}}{d_c^{1/3}} s(r)^{1/3} \tag{6}$$

valid for circular microelectrodes [18]. This calculated distribution is plotted in Fig. 5(a). It is seen that both experimental and theoretical distributions exhibit a pronounced minimum and have a similar form; also the order of magnitude of  $k_d'$  is more or less the same in both.

#### 4. Discussion

Table 1 gives correlations obtained by different authors for the overall mass transfer to a fixed disc in a confinement situation ( $C_r$  or  $C_v$ , or both). Each correlation contains  $R_T$  as the characteristic dimension. As Expression 3, established in the present work, remains unchanged when H is replaced by  $R_F$ , it can be compared with these correlations. Such a comparison is made in Fig. 6.

There is a disagreement between Correlation 3 and the correlations of Colton *et al.* [10]. However, it is important to note that the experimental correlations were obtained for the following situation: the disc was a fixed dialysis membrane in contact with a liquid swirled by an impeller rotating in a plane parallel to the membrane. Such a situation obviously differs from that of the present case. Regarding the theoretical correlation of Matsuda [8], it was established for the case of a fixed disc located in uniformly, infinite liquid; no domain of validity is known.

The correlations given in Table 2 correspond to the case of a confined rotating disc; they were established from previously uncorrelated data [7], or obtained, by analogy, from experimental [2] and theoretical [1, 2] momentum transfer results. It is noticeable that the electrochemical determination of liquid-solid mass transfer coefficients seems to have been used only in



Fig. 7. Mass transfer correlations for transfer to the rotating disc. (A) Comparison with the literature; (B) comparison for the laminar flow regime. Key for A: (a) correlation 3, this work, (b) [7], (c) [2] theoretical, (d) [1], (e) [21], (f) Equation 47 of [2] empirical, (g) Equation 46 of [2] empirical. Key for B: (a) present work, Equation 2; (b) Lehmkuhl and Hudson [7].

the present work. All the correlations of Table 2 use  $R_{\rm T}$  as the characteristic dimension.

Figure 7(A) shows that Correlation 3 agrees sufficiently well with the theoretical solutions of Soo [1] and Grunow [21], but these are only valid for  $Re_{R_T}$  higher than  $3 \times 10^5$ . For the laminar flow range, only the results of Lehmkuhl and Hudson [7] are known.

The mass transfer data presented in [7] were plotted using H as the characteristic dimension, that is, using  $\overline{Sh}_{\rm H}$  and  $Re_{\rm H}$  as the variable dimensionless numbers. They were correlated here by a unique correlation plotted in Fig. 7(B). The results of the present work described by Equation 2 are also plotted under the same form in this figure. If  $R_{\rm T}$  is introduced as the characteristic dimension, the results of [7] lead to a correlation for each  $C_{\rm v}$  value. The correlations



Fig. 8. Mass transfer at an enclosed rotating disc. Comparison with the Levich equation.

corresponding to the extreme values of  $C_v$  are plotted in Fig. 7(A). If it is taken into account that the maximum value  $C_r = 0.58$  used in [7] is much smaller than that (0.99) corresponding to the experiments of the present work, it may be concluded from Fig. 7(A) that the agreement between Correlation 3 and the results of Lehmkuhl *et al.* is satisfactory.

The comparison (Fig. 7(B)) made with the results of Lehmkuhl and Hudson [7] is interesting because the measurement methods were different in both cases. In Fig. 7(B) the deviations are noticeable at low values of  $Re_{\rm H}$ , which effectively correspond to small mass transfer flux densities, that is, to situations in which the dissolution method is imprecise.

The results of the present work show how the ratio



Fig. 9. Mass transfer at an enclosed rotating disc. Comparison with the Levich equation and empirical correlations of [16]. Key: (a) coefficient 0.84, present work; (b) coefficient 0.62, Levich [22]; (c) coefficient 0.47, Langlois [16].

 $Sh_{R_T}/Sh_{R_T}^\circ$  varies with Reynolds number  $(Sh_{R_T}$  is the Sherwood number corresponding to a confined rotating disc, and  $\overline{Sh}_{R_T}^\circ$  the Sherwood number which would be obtained for the same disc rotating at the same velocity in an infinite medium). In the laminar flow regime,  $\overline{Sh}_{R_T}^\circ$  is given theoretically by the Levich equation [22]. Figure 8 gives the variations of  $\overline{Sh}_{R_T}/\overline{Sh}_{R_T}^\circ$ with  $Re_{R_T}$  and shows that this ratio lies between 1 and 2. It was not possible to quantify the influence of  $C_v$ .

Further analysis may be based on the experimental variations of  $\overline{Sh}_{R_T}Sc^{-1/3}$  with  $Re_{R_T}^{1/2}$ . According to Expression 3, the representation would be linear with a multiplicative coefficient of 0.84. Figure 9 compares Equation 3 with the Levich equation (multiplicative coefficient of 0.62) and with the equation obtained by Langlois *et al.* [16] for  $70 \leq Re_{R_T} \leq 300$  and for mass transfer to a disc rotating at a small distance from a very porous disc ( $\epsilon$ =0.97) inserted in a wall (multiplicative coefficient of 0.47). Expression 3 seems to be satisfied mainly at the higher rotation velocities. In contrast, at small  $Re_{R_T}$ , the wall effects are small and the results are below the line representing the Levich equation, as in [16].

## 5. Conclusion

The experimental mass transfer study was limited to a small Reynolds number domain. It allowed comparison of electrochemically measured mass transfer coefficients with correlations established by other authors using less precise methods. Flow schemes proposed in other works are shown to be in agreement with experimental distributions of the local shear stress over the fixed disc.

Satisfactory empirical correlations are established. In the conditions used, the ratio  $H/R_{\rm T}$  influenced the transfer to the fixed disc.

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